was calculated in the usual way as $D_2\rho u_2/\mu$. Air was assumed to behave as an ideal gas at the low pressures employed in this work.

DISCUSSION AND CONCLUSIONS

The agreement between the measured orifice coefficients according to the American Society of Mechanical Engineers report and those determined by use of the momentum balance is considered quite good. It would be desirable in a more complete study to try other ratios of orifice to pipe diameter. However it is believed that the agreement in this experiment demonstrates the applicability of the momentum balance and the assumptions employed. The study shows rather graphically why the orifice coefficient is in the neighborhood of 0.6.

If an orifice is to be used to measure gas flow and there is no convenient way to make a calibration, the technique of measuring a few pressures on the upstream face of the orifice plate should prove useful. The most serious drawback would be the mechanical problem of making pressure taps close to the orifice opening. However this problem can be solved as shown here.

NOTATION

A = area
D = diameter
F = force
g_c = conversion factor = 32.17 (lb.-mass/lb.-force) (ft./sec.^{2})
K = orifice discharge coefficient
m = ratio of net force in actual case to that in idealized case
P = pressure
R = radius
u = velocity
μ = viscosity

Subscripts

1 = upstream position
2 = orifice opening
3 = vena contracta

LITERATURE CITED

1. Am. Soc. Mech. Engrs., "Fluid Meters, Their Theory and Application," 4 ed. (1937); Am. Soc. Mech. Engrs., "Flow Measurement," Part 5 (1940).

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The Mechanics of Vertical Moving Fluidized Systems: IV. Application to Batch-Fluidized Systems with Mixed Particle Sizes

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The present paper extends the previous investigations from this laboratory on ideal fluidized systems to a system which is somewhat nonideal. Mixtures of different but well-defined glass spheres are fluidized by water to ascertain whether the principles developed for a single particle size still hold. The analysis indicates that the ideal prediction method gives a reasonable representation of the batch-expansion curves for mixed sizes.

In a previous publication from this laboratory (3) a detailed theoretical analysis was presented for predicting the behavior of all types of vertical moving fluidized systems. The basic postulate of this development was the proposal that a simple unique relationship exists between the slip velocity and the holdup for any system. The slip velocity is defined as the relative velocity between the particles and the fluid and can be represented mathematically as

$$V_s = \text{slip velocity} = \frac{V_f'}{\epsilon} - \frac{V_d'}{1-\epsilon} \qquad (1)$$

In accordance with the theory, the holdup can be calculated for any mode of fluidization once the relationship between the slip velocity and the holdup has been determined. Figure 1 gives a typical relationship.

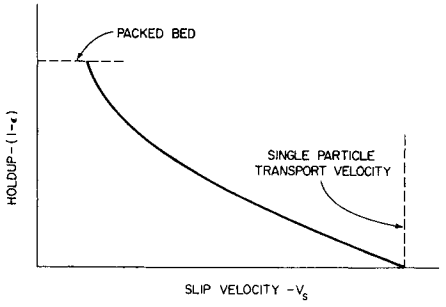

Fig. 1. Typical holdup slip velocity for an ideal batch fluidized bed.

The generalized theory has however been proved only for ideal systems (*1, 11, 12, 14*). An ideal system is one of uniform rigid spheres fluidized with a liquid having a density not too different from that of the particles. Such a system was termed *particulate fluidization* by Wilhelm and Kwauk (*16*).

The present investigation was undertaken to study the behavior of a

TABLE 1. PHYSICAL PROPERTIES OF GLASS SPHERES USED

Particle	U.S. screen	Average diameter, in.	Average deviation from average diameter, %	Density, lb./cu. ft.
1	40	0.0183	4.1	157.6
3	60	0.0106	4.5	155.2
6	170	0.00382	3.8	152.0
7	140	0.00475	4.9	154.0
8	80	0.00752	3.5	153.9

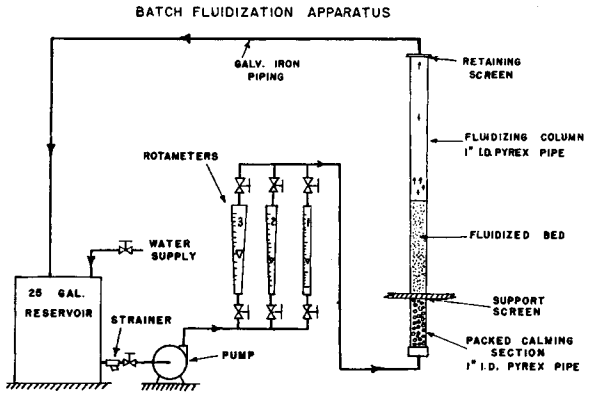

Fig. 2. Schematic diagram of fludization apparatus.

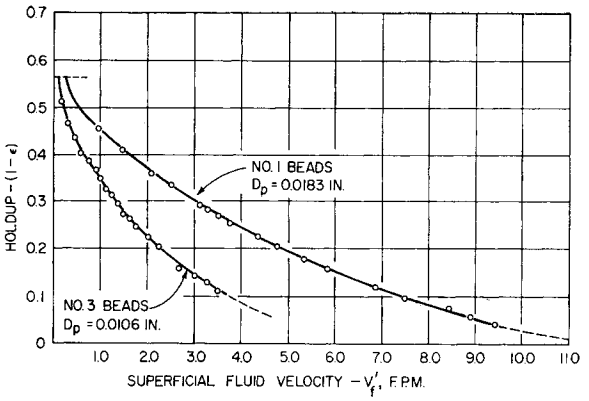

Fig. 4. Batch expansion curves for glass beads 1 and 3 in water.

batch or nonfed bottom restrained fluidized system which was somewhat nonideal, the nonideality occurring because of nonuniform particle size. To develop the necessary principles, mixed beds consisting of two or three distinct measurable particle sizes rather than a spectrum of sizes were used. The experimental work was intended to provide basic information on a batch system which could be used for a possible extension or modification of the generalized theory to include nonideal systems.

LITERATURE SURVEY

Some work has been done previously by other investigators dealing with fluidized beds with mixed particle sizes. These data however are few and in some cases conflicting. No previous systematic study of the effect of particle-size distribution on the batch expansion curves (Figure 1) has apparently been made.

Lewis, Gilliland, and Bauer (*6*) and Morse (*10*) report the use of an average particle diameter in correlating data from batch expansion with mixed sizes along with uniform particle data. Such correlations are in the form of modified-drag-coefficient–Reynolds-number plots. However no numerical size distributions within the bed were reported.

Mixtures of various-sized particles have been separated by passing a fluid upward through a suspension of mixed size particles, effecting a transport of the smaller-sized particles from the mixture. In a batch-fluidized system McCune and Wilhelm (*7*), Lewis and Bowerman (*5*), and Richardson and Zaki (*13*) observed a size distribution within the bed, the fines being removed to the top of the bed. Richardson and Zaki further found that for one experiment with a binary mixture of particle sizes the holdup could be calculated as the weighted sum of the holdups of each individual particle size.

Mertes and Rhodes (*8, 9*) mention briefly the possibility of attacking the problem of holdup-slip–velocity relationship for mixtures of sizes but do not present any experimental work.

More recently Furukawa and Tsutoma (*2*) have performed miscibility experiments for fluidized beds with binary mixtures of various-sized sand and charcoal. They observed size separation within the fluidized bed and the formation of two layers consisting of each component of the mixture, respectively. The reported separation, however, was found to be only a partial one. Their work did not include the effect of mixed sizes on the actual batch expansion curves.

The recent work of Andrieu (*4*) is probably the most complete and thorough available at the present time on segregation of solids.

Previous work is not sufficiently complete to permit the prediction of batch expansion curves but does indicate the possibility of size separation.

EXPERIMENTAL

Fluidization Apparatus

The fluidization apparatus consisted of a glass fluidizing column, a pump, a water reservoir, rotameters, and interconnecting piping as shown schematically in Figure 2.

The fluidizing column was constructed of 1-in. Pyrex pipe, at either end of which was a retaining screen made of U.S. 170 bronze-wire cloth. Below the fluidizing column was a 12-in. packed section which minimized velocity gradients at the inlet to the fluidizing column. The pump was a three-stage centrifugal one.

Glass Spheres

Binary and ternary mixtures of particle sizes were made from known quantities of glass spheres, each having a uniform diameter. These were obtained by a careful screening of glass beads. Table 1 lists the pertinent physical properties of the particles used.

By means of a calibrated micrometer eyepiece, the beads were examined microscopically to check the sphericity and to

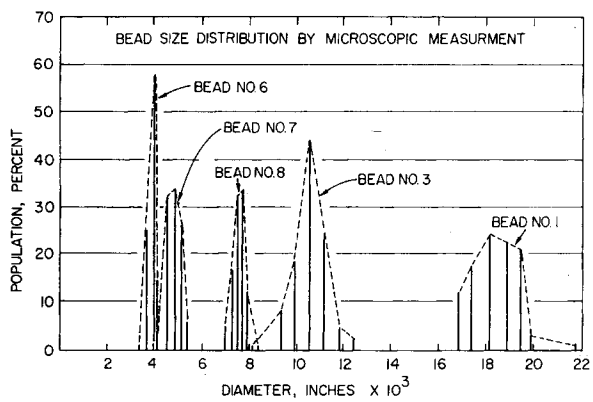

Fig. 3. Graphical representation of bead-size distribution as determined by microscopic measurement.

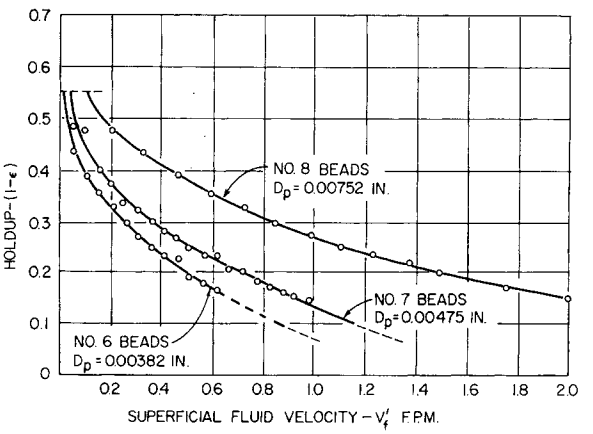

Fig. 5. Batch expansion curves for glass beads 6, 7, and 8 in water.

Table 2. Particle Mixture Used in Present Experiments

	Particle mixture A	Particle mixture B	D_a/D_b	Quantity in mixture, % B
I	Particle 1	Particle 3	1.73	50
II	Particle 1	Particle 3	1.73	80
III	Particle 8	Particle 7	1.58	33.3
IV	Particle 8	Particle 7	1.58	50
V	Particle 7	Particle 6	1.24	33.3
VI	Particle 7	Particle 6	1.24	70
VII	Particle 3, 8 and 7		2.23/1.58/1.0	33.3% of each

determine size distribution. The average diameter and size distribution were obtained from measurement of a random sample of one hundred beads. The spectra of size distribution is shown graphically in Figure 3. Although a spread of diameters is seen in this diagram, the average percentage deviation from the average particle diameter is less than 5%. This narrow size distribution is the closest approximation to uniformly sized beads that can be approached by standard screening techniques.

The density of each bead size was determined by water displacement.

Glass-Bead Coloring Procedure

To observe the extent of the segregation during fluidization of binary and ternary mixtures, one size of beads in a mixture was colored by a process originally proposed for dyeing glass fibers (*15*). The fibers are heated with various ionic solutions, and supposedly the ions are absorbed in the glass by a base exchange of the solution ions with the alkali and alkaline earth metals in the glass. It was found that a blue coloration, produced by the formation of Prussian blue, was most easily reproduced on the beads. Density measurements after coloration showed that the coloring process had no significant effect upon the bead density.

Fluidization Procedure

Each uniform size of glass beads (No. 1, 3, 6, 7, and 8) was fluidized separately to determine the individual batch expansion curves. The procedure used in the fluidization of the individual-sized beads was as follows:

1. A given weight of beads was fluidized in the column until the bed had expanded to the full height of the column.

2. The water rate was decreased incrementally, with the height of the fluidized bed and the fluid rate being measured after each increment. Time was allowed for the system to reach equilibrium at each fluid rate.

3. The water temperature was maintained at 22° to 23°C. throughout each run.

A similar experimental technique was used in the fluidization of the mixed particle sizes. In this latter case, however, each bead size was of a different color, and the heights of the individual segments of the bed (blue and white layers) were measured in addition to over-all height.

RESULTS

Individual Batch Curves

The batch-expansion curves obtained for each individual bead size, shown in Figures 4 and 5, are presented in terms of superficial fluid velocity, rather than slip velocity, to facilitate the prediction of the batch expansion curves for binary and ternary mixtures, as will be described later.

Size Separation in Mixtures of Sizes

The six binary mixtures and one ternary mixture which were fluidized in this investigation are presented in Table 2.

Two types of size separation were observed in the experiments with mixed sizes. First, in the case where the beads were of widely different diameters (I, II, III, IV, and VII), a very sharp size separation was noted. When steady state was reached, two distinct layers were seen with a very clearly defined interface between the layers. This layer formation was easily observed, since each size of bead was a different color. The separation into sizes was essentially complete, with few if any particles of one size detectable in the zone of the other.

In the second case (V and VI,) where the beads were of nearly the same diameter, only a partial separation into layers was observed. In this case a transition region existed between the two layers. The height of this transition region varied with the fluid velocity. As the fluid velocity was decreased, the upper layer became smaller, while the height of the upper interface remained almost constant. Thus the fraction of the bed height occupied by the transition region increased with decreasing fluid velocity. The behavior of this fluidized system suggests strongly the analogous behavior of liquid mixtures which may be partially or completely immiscible and in which the degree of miscibility is determined by the temperature (here in the form of fluid velocity or energy).

Prediction of Batch Expansion Curves for Mixtures of Sizes

If a perfect size separation occurs in a fluidized bed of mixed particle sizes of the same density, the total height of the bed will be the sum of the heights of the individual particle-size layers. The height of each layer is given by

$$L_i = \frac{W_i}{\rho_i} \cdot \frac{1}{A} \cdot \frac{1}{(1-\epsilon_i)} \qquad (2)$$

and the total holdup defined as

$$(1-\epsilon_t) = \frac{W_t}{\rho_t} \cdot \frac{1}{A} \cdot \frac{1}{L_t} \qquad (3)$$

In terms of individual particle-layer weights the holdup thus becomes

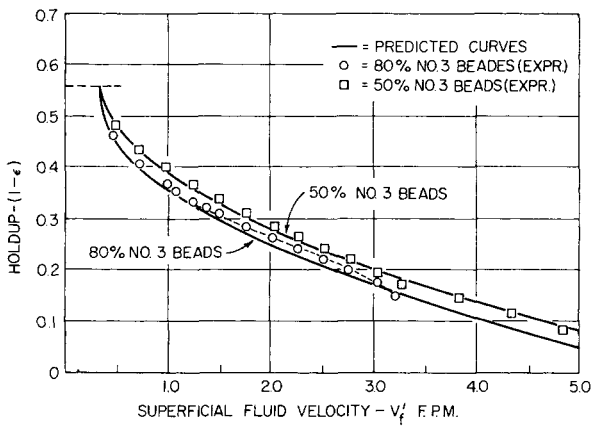

Fig. 6. Comparison of experimental data with predicted batch expansion curves for binary mixtures of glass beads 1 and 3.

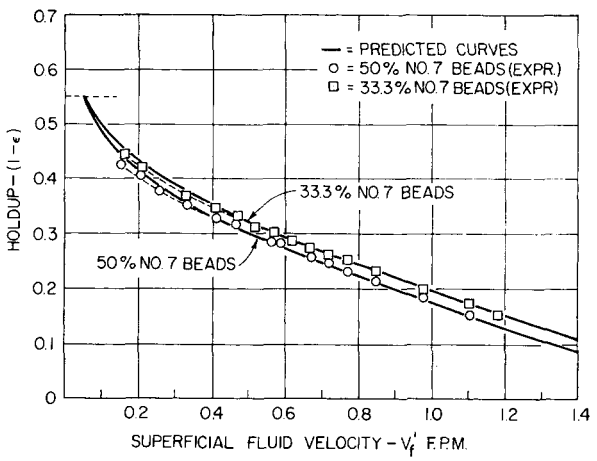

Fig. 7. Comparison of experimental data with predicted batch expansion curves for binary mixtures of glass beads 7 and 8.

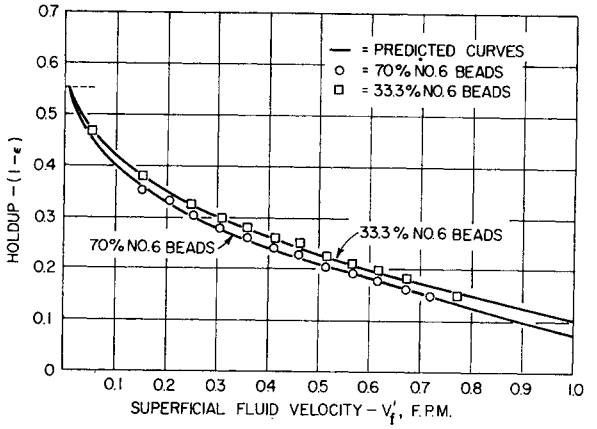

Fig. 8. Comparison of experimental data with predicted batch expansion curves for binary mixtures of glass beads 6 and 7.

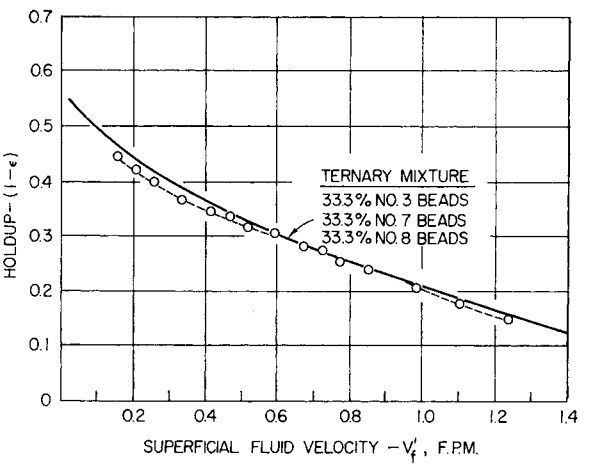

Fig. 9. Comparison of experimental data with predicted batch expansion curves for a ternary mixture of glass beads 3, 7, and 8.

$$(1-\epsilon_t) = \frac{1}{\frac{X_1}{1-\epsilon_1} + \frac{X_2}{1-\epsilon_2} + \ldots + \frac{X_i}{1-\epsilon_i}} \qquad (4)$$

$$= \frac{1}{\sum_{n=1}^{i} \frac{X_n}{1-\epsilon_n}}$$

The comparison between experimental holdup-fluid velocity curves for mixed particle sizes and those calculated by means of Equation (4) and the experimentally determined slip velocity-holdup curves measured for the individual particle sizes is shown in Figures 6 to 9. For the most part the experimental expansion data give excellent agreement with the theoretical curves as predicted from the single-size–particle data. The error calculated for the majority of the points is of the order of 3% or less with the maximum error being 7 to 8%. It is seen that in some cases the experimental data curves contain some discontinuities in the form of bumps or dips, which are believed to be the result of nonidealities in the individual layers of the fluidized bed. Such nonidealities can produce higher or lower holdups in each layer than were observed for a fluidized bed of uniform particle size.

An important result is that the calculated and experimental curves agree extremely well for the partially segregated beds having closely sized particles.

DISCUSSION

The nonidealities of the individual layers which produce in some of the mixtures deviations from the over-all predicted curves may be the result of one of two factors. First, the individual layers of the mixtures may behave as partially restrained systems. When the uniformly sized beads are fluidized, there is no restriction at the top of the bed, while at the bottom there is a fixed restraint in the form of the bed support screen. In a two- or three-layer mixture, however, the lower layer is partially restrained at the top by the presence of the upper layer, and this restraint can cause a higher holdup in the lower region. In addition, the upper layer is no longer totally restricted at the bottom, since the only restriction is the variable height of the lower layer. Second, there may be some degree of mixing between layers, causing a mixture of sizes within each individual holdup from the ideal case.

In spite of these nonidealities, in most cases the ideal prediction curve, Equation (4), gives excellent agreement with the experimental data. The actual random motion which takes place on the individual particle level in a fluidized bed is an extremely complex phenomenon, and any prediction method must be a simplification of the actual process. Recognizing this fact, one must make the general conclusion that the ideal prediction method gives a reasonable representation of the batch expansion curves for mixed sizes. As a result it is possible to predict the holdup curves for mixed particle sizes through the use of Equation (4) and any one of the several correlations available for the batch expansion curve.

NOTATION

A = cross-section area of fluidizing column
L = height of fluidized bed
V_f' = superficial fluid velocity
V_d' = superficial net particle velocity through fluidized bed
V_s = slip velocity
W = weight of particles having a diameter D_i
X = weight fraction
ϵ = void fraction of fluidized bed
$(1-\epsilon)$ = holdup of fluidized bed
ρ_i = density of particles having a diameter D_i

Subscript

n = nth particle diameter in the mixture

LITERATURE CITED

1. Flinn, D. R., M. S. thesis, Princeton Univ., Princeton, N. J. (1954).
2. Furukawa, J., and O. Tsutoma, *Ind. Eng. Chem.*, **50**, 821 (1958).
3. Lapidus, Leon, and J. C. Elgin, *A.I.Ch.E. Journal*, **3**, 63 (1957).
4. Leva, Max, Personal communication.
5. Lewis, E. W., and E. W. Bowerman, *Chem. Eng. Progr.*, **48**, 603 (1952).
6. Lewis, W. K., E. R. Gilliland, and W. C. Bauer, *Ind. Eng. Chem.*, **41**, 1104 (1949).
7. McCune, L. K., and R. H. Wilhelm, *ibid.*, 1124 (1949).
8. Mertes, T. S., and H. B. Rhodes, "Fluid-Particle Behavior," Sun Oil Company, Marcus Hook, Pa.
9. ———, *Chem. Eng. Progr.*, **51**, 429, 517 (1955).
10. Morse, R. D., *Ind. Eng. Chem.*, **41**, 1117 (1949).
11. Price, B. G., Leon Lapidus, and J. C. Elgin, *A.I.Ch.E. Journal*, **5**, 93 (1959).
12. Quinn, J., Ph.D. thesis, Princeton Univ., Princeton, N. J. (1958).
13. Richardson, J. F., and W. N. Zaki, *Trans. Inst. Chem. Eng., (London)*, **32**, 35 (1954).
14. Struve, D. L., Leon Lapidus, and J. C. Elgin, *Can. J. Chem. Eng.*, **36**, 141 (1958).
15. *U.S. Patent 2,245,783* (1941).
16. Wilhelm, R. H., and Mooson Kwauk, *Chem. Eng. Progr.*, **44**, 201 (1948).

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